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# A UNILATERAL REPRESENTATION FOR AUTOREGRESSIVE RANDOM FIELD MODELS

P. R. Thrift
~~Computer Vision~~ Laboratory
Computer Science Center
University of Maryland
College Park, MD 20742

ABSTRACT

This paper discusses autoregressive random field (ARF) models and derives a unilateral representation driven by uncorrelated noise.



page - A -

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## 1. Introduction

In this paper we shall deal with a subset of stationary (wide sense) processes with absolutely continuous spectral distributions which are rational functions of the two quantities $e^{i\theta_1}, e^{i\theta_2}$. More precisely we shall study the process $X_{[m,n]} \in R^d$, $[m,n] \in \mathbb{Z} \times \mathbb{Z}$ on an infinite lattice, with covariance structure

$$E(X_{[m+s,n+t]} X^*_{[m,n]}) =$$

$$\frac{1}{4\pi^2} \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-is\theta_1 - it\theta_2} f(\theta_1,\theta_2)\, d\theta_1 d\theta_2, \tag{1.1}$$

and zero mean.

We assume $f(\theta_1,\theta_2)^{-1}$ exists and is finite at every $(\theta_1,\theta_2)$, and

$$f(\theta_1,\theta_2)^{-1} = (a_{[0,0]} + \sum_{[m_1,m_2] \in N^p} a_{[m_1,m_2]} \cos(m_1\theta_1 + m_2\theta_2)) \tag{1.2}$$

$a_{[0,0]}, a_{[m_1,m_2]}$ are p x p matrices satisfying $a^*_{[s,t]} = a_{[-s,-t]}$. $V^*$ is the complex conjugate transpose of the vector V. $N^p$ denotes the deleted p x p neighborhood of [0,0], that is,

$$\{[m_1,m_2] : |m_1| \le p, |m_2| \le p, [m_1,m_2] \ne [0,0]\}.$$

Models of this type have been used as models of texture images [1,2]. In the case where $X_{[\cdot,\cdot]}$ is a Gaussian process, it can be shown [3] that $X_{[\cdot,\cdot]}$ is a Gauss-Markov process with respect to $N^p$; that is,

$$\mathrm{Prob}(X_{[m,n]} | X_{[s,t]}, [s,t] \neq [m,n]) =$$

$$\mathrm{Prob}(X_{[m,n]} | X_{[s,t]}, [s,t] \in [m,n] + N^P) \tag{1.3}$$

In fact, the process with spectral density $f(\theta_1, \theta_2)$ satisfies the conditional model

$$E(X_{[m,n]} | X_{[s,t]}, [s,t] \neq [m,n]) =$$

$$-a^{-1}_{[0,0]} \left( \sum_{[m_1,m_2] \in N^P} a_{[m_1,m_2]} X_{[m-m_1,n-m_2]} \right) \tag{1.4}$$

Conditional models of this type have been found useful in the modeling of spatial patterns [7]. It is also known (see, for example, page 26 of [7]) that no finite one-sided representation for this model exists of the type (with S finite subset of $\mathbb{Z} \times \mathbb{Z}$)

$$b_{[0,0]} X_{[m,n]} + \sum_{[m_1,m_2] \in S} b_{[m_1,m_2]} X_{[m-m_1,n-m_2]} = Z_{[m,n]}$$

where $Z_{[m,n]}$ is a process of uncorrelated noise.

The purpose of this paper is to show that the collection of spectral representations of the process $X_{[\cdot,\cdot]}$ along one of the coordinates is representable as a one-sided finite order "time series" model along the other coordinate. Thus, in this sense it is seen that all ARF's have a "causal" representation.

This method of producing a one-sided representation can be contrasted with the so-called NSHP (non-symmetric half plane) representation of [3] and [6].

## 2. A Unilateral Representation

We consider the process $X_{[.,.]}$ with spectral density

$$f(\theta_1,\theta_2)=[a_{[0,0]}+\sum_{[m_1,m_2]\in N^p} a_{[m_1,m_2]}\cos(m_1\theta_1+m_2\theta_2)]^{-1}, \tag{2.1}$$

which is a p-th order autoregressive process.

Let $z=e^{-i\theta_1}$, $w=e^{-i\theta_2}$, and rewrite the above equality as

$$f(\theta_1,\theta_2)^{-1}=a_0(w)+a_1(w)z+\ldots+a_p(w)z^p$$
$$+a_1^*(w)z^{-1}+\ldots+a_p^*(w)z^{-p}.$$

For each fixed w we can consider $f(\theta_1,\theta_2)$ as a spectral density in $\theta_1$. We next produce a causal factorization of $f(\theta_1,\theta_2)$ in the form

$$f(\theta_1,\theta_2)^{-1}= \tag{2.2}$$
$$[c_0^*(w)+c_1^*(w)z^{-1}+\ldots+c_p^*(w)z^{-p}][c_0(w)+c_1(w)z+\ldots+c_p(w)z^p],$$

where, for each $w=e^{-i\theta_2}$, $c_0(w)+c_1(w)\xi+\ldots+c_p(w)\xi^p$ has no roots inside and on the complex unit circle $|\xi|=1$. ([4], page 65).

We next consider the spectral representation of the process $X_{[.,.]}$ along the second coordinate:

$$X_{[n,m]}=\int_{-\pi}^{\pi}e^{im\theta}dY_n(\theta), \tag{2.3}$$

where $Y_n(\theta)$ is the spectral representation of the process $X_{[n,.]}$. ([5], page 481).

Next expand each of $c_0(w),\ldots,c_p(w)$ in a Fourier expansion

$$c_j(w)=\sum_{k=-\infty}^{\infty} e^{ik\theta_2} \hat{c}_{[j,k]}.$$

Then ([4], page 61) the process satisfies the autoregression

$$\sum_{j=0}^{p} \sum_{k=-\infty}^{\infty} \hat{c}_{[j,k]} X_{[n-j,m+k]} = Z_{[n,m]} \tag{2.4}$$

where $Z_{[.,.]}$ is an uncorrelated white noise process. Let $W_n(\theta)$ be the spectral representation of the process $Z_{[n,.]}$:

$$Z_{[n,m]} = \int_{-\pi}^{\pi} e^{im\theta} dW_n(\theta). \tag{2.5}$$

We conclude with the following

Theorem: Let $\{Y_n(\theta)\}$, $\{W_n(\theta)\}$ be the spectral representations of the processes defined above. They satisfy the stochastic differential equation

$$\sum_{k=0}^{p} c_k(e^{-i\theta}) dY_{n-k}(\theta) = dW_n(\theta) \tag{2.6}$$

Proof: In the above autoregressive representation we substitute the spectral integrals and get (after combining terms)

$$\forall m: \int_{-\pi}^{\pi} \{\sum_{j=0}^{p} \sum_{k=-\infty}^{\infty} \hat{c}_{[j,k]} e^{i(m+k)\theta} dY_{n-j}(\theta) - e^{im\theta} dW_n(\theta)\} = 0$$

Factoring out $e^{im\theta}$ we have

$$\forall m: \int_{-\pi}^{\pi} e^{im\theta} \{\sum_{j=0}^{p} \hat{c}_j(e^{-i\theta}) dY_{n-j}(\theta) - dW_n(\theta)\} = 0$$

As any continuous function $f(\theta), \theta \in [-\pi,\pi)$ can be approximated in mean square by linear combinations of $e^{im\theta}$, the result follows.

## 3. The finite version.

The above calculation can be carried out in the case where we have a finite number of values

$$X_{[n,0]}, \ldots\, X_{[n,M-1]}$$

in the vertical direction.

Let $\psi_M = e^{2\pi i/M}$. The finite versions of the above spectral representations are as follows:

$$X_{[n,m]} = \sum_{k=0}^{M-1} \psi_M^{km} Y(n,k) \tag{3.1}$$

or

$$\Delta Y(n,k) = \frac{1}{M} \sum_{j=0}^{M-1} \psi_M^{-km} X_{[n,j]}. \tag{3.2}$$

That is, $\Delta Y(n,\cdot)$ is the finite Fourier transform of the data $X_{[n,.]}$. Similarly

$$\Delta W(n,k) = \frac{1}{M} \sum_{j=0}^{M-1} \psi_M^{-km} Z_{[n,j]}. \tag{3.3}$$

The finite analogue of the above Theorem is

$$\sum_{k=0}^{M-1} \psi_M^{mk} \left\{ \sum_{j=0}^{p} b_j (\psi_M^{-k}) \Delta Y(n-j,k) - \Delta W(n,k) \right\} = 0$$

concluding with

$$\sum_{j=0}^{p} b_j (\psi_M^{-k}) \Delta Y(n-j,k) = \Delta W(n,k). \tag{3.4}$$

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